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LETTER TO THE EDITOR

Asymptotic realization of the criterion for quantum integrability of a boson system with dynamical symmetry

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Abstract. We investigate the energy-level statistics in dependence on the boson number and the underlying classical motion for a system of collective states of zero angular momentum in γ-soft nuclei described in the framework of the O(6) dynamical symmetry of the interacting boson model. This presents a relatively complex test case for relations between classical regular/chaos, dynamical symmetry and energy-level statistics. The classical limit is integrable due to an additional constant of motion P_γ, but the corresponding quantal energy-level statistics in the range of realistic parametrization is close to Gaussian orthogonal ensemble statistics. However, the boson number N_B, which plays the role of Planck constant in semiclassical approximation, appears as a control parameter of quantum chaos, and the energy level statistics asymptotically approaches Poisson statistics with increasing boson number, i.e. with decreasing volume of the unit cell of quantum space.

The investigation of the manifestations of classical chaos in quantum systems have been initiated by a prediction that quantal energy spectrum of a dynamical system consists of a regular and chaotic part (Percival 1973). Bohigas *et al* (1984) have formulated a conjecture that a quantal energy spectrum is characterized by Poisson statistics when the corresponding classical system is regular and by Gaussian orthogonal ensemble (GOE) statistics when it is chaotic. This conjecture was confirmed in studies of low-dimensional systems (Seligman *et al* 1985, Wintgen and Friedrich 1987, Meredith *et al* 1988), although some exceptions are known (Cheon and Cohen 1989, Balasz and Voros 1986, Casati *et al* 1985, Berry and Tabor 1977, Eckhardt 1988). For Δ_3 statistics there appear systematic deviations from GOE due to saturation which was explained using semiclassical arguments (Berry 1985). On the other hand, attention has been paid to the investigation of connections between dynamical symmetry and chaos (Robnik 1981, Zhang *et al* 1988, 1989, Alhassid *et al* 1990, Paar and Vorkapić 1988, 1990). Zhang *et al* (1989) have formulated a framework with a criterion for quantum integrability associated with dynamical symmetry. In this framework, the chaotic motion is associated with breaking of dynamical symmetry.

A convenient model to study these concepts is the interacting boson model (IBM) (Arima and Iachello 1975) which describes the structure of nuclei. This model has a real physical background: it describes the low-lying nuclear phenomenology accounting for collectivity. Furthermore, it has a microscopic basis and provides a framework for algebraic description including dynamical symmetries. In IBM one considers a system of N_B bosons that can occupy six levels, namely an s level (with angular momentum $L=0$) and a five-fold degenerate d_μ ($L=2$) level, interacting through a Hamiltonian

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that can be expressed in terms of 36 generators s^+s, $d^+_\mu s$, s^+d_μ and $d^+_\mu\tilde{d}_\nu$ of the group U(6) (Arima and Iachello 1975, 1987). Here, μ and ν denote the angular momentum projections and $\tilde{d}_\nu=(-1)^\nu d_{-\nu}$. The chaotic features of IBM have been previously investigated (Paar and Vorkapić 1988, 1990, Alhassid *et al* 1990). It was found that near the SU(3) and O(6) dynamical symmetries of IBM the fluctuation behaviour of the states with angular momentum $L\geqslant 2$ is close to Poisson statistics and changes gradually to the GOE statistics as the interaction strength in IBM Hamiltonian moves away from these dynamical symmetries (Alhassid *et al* 1990). Although the underlying classical Hamiltonian was not investigated, Monte Carlo calculations applied to boson condensates gave an evidence for classical motion underlying the SU(3) and O(6) dynamical symmetries (Alhassid *et al* 1990). These results were in agreement with the criterion by Zhang *et al* (1989) in conjunction with the conjecture by Bohigas *et al* (1984). On the other hand, a recent investigation of the states with angular momentum zero in rotational nuclei described by the SU(3) dynamical symmetry of IBM has revealed a case where both the criterion by Zhang *et al* and the conjecture by Bohigas *et al* are violated (Paar *et al* 1991). In this paper we investigate the O(6) dynamical symmetry of IBM, which turns out to have a different chaotic behaviour.

The Hamiltonian for the O(6) limit of IBM can be presented in the form (Dieperink 1982)

$$H^{O(6)}=\frac{1}{4}\frac{\varkappa_0}{N_B-1}P^+\cdot P+\frac{\varkappa_5}{N_B-1}\hat{C}_5+\frac{\varkappa_1}{N_B-1}L^{(1)}\cdot L^{(1)} \tag{1}$$

with the O(6) pairing operator

$$P^+\cdot P=(d^+\cdot d^+-s^+s^+)(\tilde{d}\cdot\tilde{d}-s\,s) \tag{2}$$

the O(5) Casimir invariant

$$\hat{C}_5=\tfrac{1}{3}\{(d^+\tilde{d})^{(3)}\cdot(d^+\tilde{d})^{(3)}+\tfrac{1}{10}L^{(1)}\cdot L^{(1)}\} \tag{3}$$

and the angular momentum operator

$$L^{(1)}_\mu=\sqrt{10}(d^+\tilde{d})^{(1)}_\mu. \tag{4}$$

The quantities $\varkappa_0$, $\varkappa_5$ and $\varkappa_1$ are the interaction strengths and N_B is the total number of s and d bosons.

The classical Hamiltonian corresponding to the quantal Hamiltonian (1) was obtained (Dieperink *et al* 1980, Ginocchio and Kirson 1980) as the coherent state expectation value of the quantum operator (1)

$$H^{O(6)}_{cl}=\langle N_B,\alpha|H^{O(6)}|N_B,\alpha\rangle/N_B \tag{5}$$

with

$$|N_B,\alpha\rangle=\frac{1}{\sqrt{N_B!}}\left\{(1-|\alpha|^2)^{1/2}s^++\sum_\mu\alpha_\mu d^+_\mu\right\}^{N_B}|0\rangle. \tag{6}$$

The resulting classical Hamiltonian is

$$H^{O(6)}_{cl}=\frac{\varkappa_0}{4}[\beta^2P^2_\beta+(1-\beta^2)^2]+\frac{\varkappa_5}{6}P^2_\gamma. \tag{7}$$

Here, β and γ are the intrinsic coordinates, having a physical meaning of quadrupole shape parameters, while P_β and P_γ are their conjugate momenta.

The classical equation of motion associated with the Hamiltonian (7) are

$$\dot{\beta} = \frac{\varkappa_0}{2} \beta^2 P_\beta \tag{8}$$

$$\dot{P}_\beta = -\frac{\varkappa_0}{2} (\beta P_\beta^2 - 2\beta(1-\beta^2)) \tag{9}$$

$$\dot{\gamma} = \frac{\varkappa_5}{3} P_\gamma \tag{10}$$

$$\dot{P}_\gamma = 0. \tag{11}$$

Due to (11), the quantity P_γ is a constant of motion. Thus, for a system (7) with two degrees of freedom we have two constants of motion, i.e. the momentum P_γ in addition

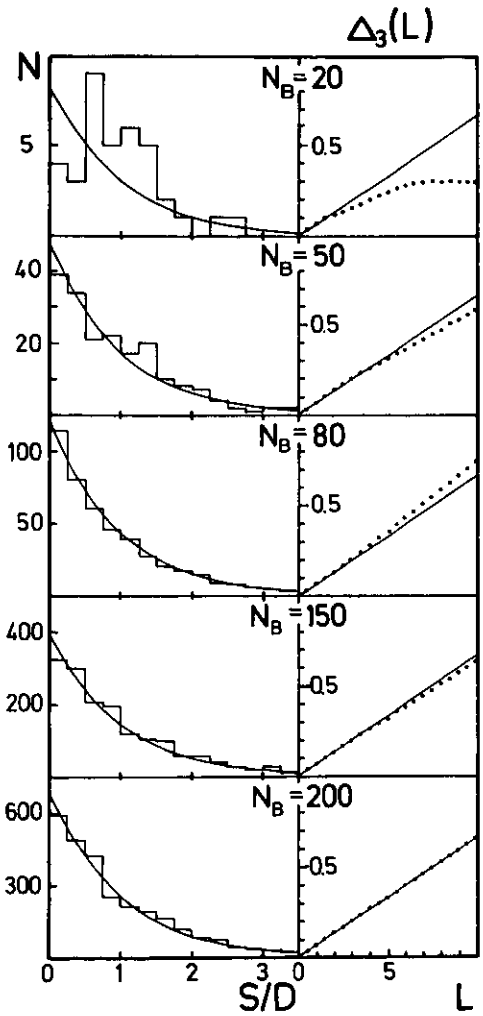

Figure 1. Nearest-neighbour spacing distribution $P(S)$ (histograms in the first column) and Dyson–Mehta Δ_3 statistics (dots in the second column) for $L^\pi = 0^+$ eigenstates of the Hamiltonian (1). The O(6) parameters are $\varkappa_0 = 0.170$, $\varkappa_5 = -1.00$ and $\varkappa_1 = -0.001$. The results are presented for several values of the total boson number N_B.

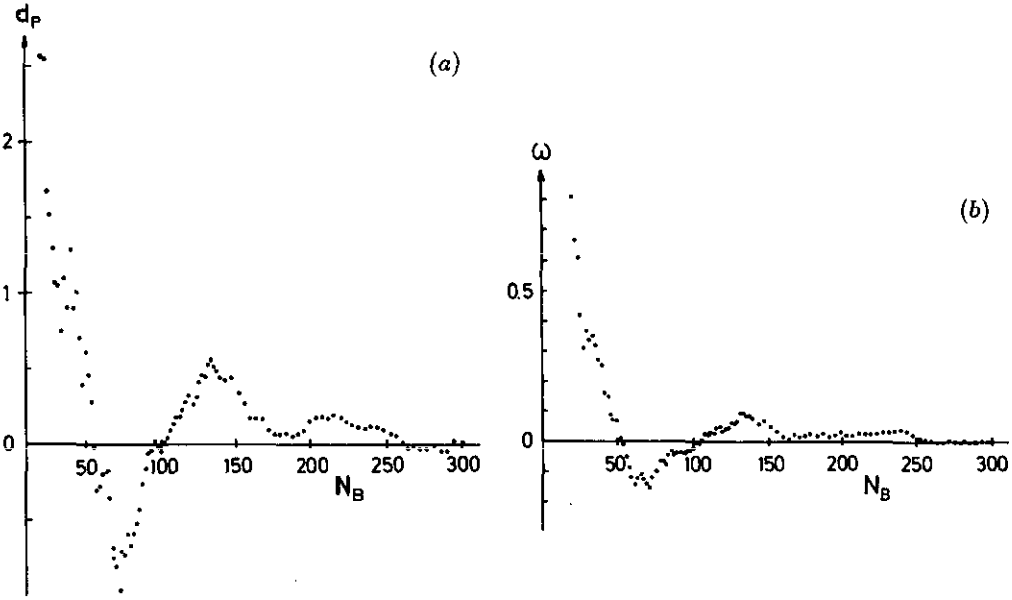

Figure 2. (*a*) Deviations of the calculated Δ_3 statistics from the $L/15$ behaviour, which corresponds to Poisson statistics, in dependence on the boson number N_B. As a measure of deviations we use $d_p = \Sigma_{2L=1}^{20} [L/15 - \Delta_3(L)]$. (*b*) Brody parameter ω for the calculated NNS distribution in dependence on the boson number N_B. The values of ω corresponding to Poisson and GOE statistics are 0 and 1, respectively.

to the energy. Consequently, the system is integrable. This result for a system with dynamical symmetry is in accordance with the criterion by Zhang *et al* (1989).

Let us now investigate the energy-level statistics for the quantum mechanical Hamiltonian (1). Being associated with an underlying regular classical motion (7), the energy-level statistics for the eigenvalues of (1) should be of Poisson type according to the conjecture by Bohigas *et al* (1984). The calculated fluctuation measures, nearest-neighbour spacing (NNS) distribution and Dyson–Mehta Δ_3 statistics, are presented in the first row of figure 1 for the boson number $N_B = 20$. As seen, they are closer to the GOE than to Poisson statistics. (For the NNS distribution the value of Brody parameter is $\omega = 0.81$.) Thus, the energy-level statistics for O(6) dynamical symmetry of IBM violates the conjecture by Bohigas *et al* (1984).

For heavy nuclei with protons and neutrons in the respective open shells the boson number is $N_B \leqslant 20$. A particularly convenient feature of IBM is that it provides an opportunity to study the behaviour of energy-level statistics as the system evolves towards the classical limit. Namely, the boson number N_B plays the role of a control parameter for semiclassical approximation, with $N_B \to \infty$ corresponding to the classical limit (Dieperink *et al* 1980, Ginocchio and Kirson 1980). Thus, $1/N_B$ plays a role of $\hbar$ in a mean field approximations (Alhassid *et al* 1990) and the volume of the unit cell in the quantum space is proportional to $1/N_B$. In the second to fifth row of figure 1 we present NNS distributions and Δ_3 statistics for increasing values of the boson number N_B. As seen, with increase of N_B the energy-level statistics shows a tendency towards Poisson statistics. However, this suppression of chaos is not monotonic. Instead, we find, irregular oscillations around the Poisson statistics as shown in figure 2. The physical significance of these oscillations around the Poisson statistics has to be explored. It is tempting, however, to conclude that the energy-level statistics asymptotically approaches the Poisson statistics in the limit of large N_B.

In conclusion, we have found the following features of O^+ states in γ-soft nuclei described by the O(6) dynamical symmetry of IBM.

(i) The underlying classical motion is integrable, which is in accordance with the criterion by Zhang *et al* (1989) on the connection between classical integrability and dynamical symmetry.

(ii) The energy-level statistics is close to GOE statistics, which is in contrast to the widely agreed conjecture by Bohigas *et al* (1984).

(iii) With decreasing volume of the unit cell of quantum space ($\sim 1/N_B$), the energy-level statistics seems to asymptotically approach the Poisson statistics. In this process there appear irregular oscillations around the Poisson fluctuations measures. This gives a hint that the conjecture by Bohigas *et al* (1984) in conjunction with the criterion by Zhang *et al* (1989), although violated in the range of realistic parameter values, is asymptotically satisfied in the limit of large N_B, i.e. of small unit cell of quantum space. This is in accordance with the general theory by Berry (1985) which gives an exact connection between energy level statistics and chaos/regularity in the semiclassical limit.

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